

# ELECTRON SCATTERING AND IMAGE CONTRAST IN ELECTRON MICROSCOPY.

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**ABSTRACT.** In the present paper, the elastic scattering cross-sections for a 60 k.v. electron beam have been calculated for several different elements from the equations of Moliere and Lenz. The variation of the scattering cross-section with the aperture angle and with the beam voltage has been considered. The inelastic scattering cross-sections for the same elements have also been estimated from Lenz equations. The values of different cross-sections have been compared and their relative importance in the production of contrast in an electron microscopic image has been estimated.

## 1. INTRODUCTION

The degree of scattering of electrons in a specimen determines the contrast in the image formed by an electron microscope. Thin metallic films are often used for enhancing the contrast of biological specimens, because the electron scattering power of the specimen is then artificially increased due to deposit of heavy atoms on its surface. It is, therefore, important to estimate the contrast to be expected for the deposit of a known thickness of metal vapour.

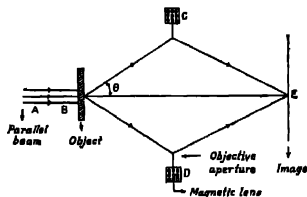


Fig. 1. Elastic cross-section due to Moliere and Lenz for 60 k.v. electrons as a function of  $\theta$  for different elements.

In figure 1, a parallel beam of electrons is incident on the specimen. The electrons are scattered in all directions and emerge out of the specimen as a divergent beam. The electrons scattered at angles less than  $\theta$  with the optic axis are allowed by the objective aperture and focussed by the lens on the image plane, while the electrons which are scattered beyond  $\theta$  are lost to the image.

The scattering by free electrons and multiple scattering are usually neglected in electron microscopic specimens which are normally very thin. The incident

electrons on encountering the specimen are mainly scattered in two different ways :

- a) elastic scattering with no energy loss, but change of direction,
- b) inelastic scattering resulting in excitation or ionisation of the atom.

The total scattering cross-section is, therefore, given by,

$$\sigma = \sigma_e + \sigma_i \quad \dots (1)$$

where,  $\sigma_e$  = elastic scattering cross-section, and  $\sigma_i$  = inelastic scattering cross-section.  $\sigma_e$  and  $\sigma_i$  have been calculated in the following sections for uranium, gold, palladium and chromium—the elements frequently used in shadow casting cross-sections for carbon have also been estimated as all biological materials consist mostly of carbon.

## II. ELASTIC SCATTERING CROSS-SECTION

Various elastic scattering formulae have been proposed to estimate the scattering cross-sections. Starting with the theory of differential cross-section of scattering by Williams (1939), Chaudhuri (1952) of this laboratory derived a formula of elastic cross-section. In his review, Von Borries (1949) favoured the formula of Moliere (1947) which is based on Thomas-Fermi atom model. But the formula derived by Lenz (1954) has been found to be in better agreement with experimental results by various investigators [Biberman (1949), Leonhard (1954), Haine and Agar (1956), Kempf and Lenz, 1956]. In this paper scattering cross-sections based on both Moliere and Lenz-equations have been calculated and compared.

### a) Elastic Scattering According to Moliere.

According to Moliere, for a beam of electrons of energy  $e\phi_R$  the differential cross-section of scattering per unit solid angle between an angle  $\theta$  and  $\theta + d\theta$  for fast electrons is given by,

$$\begin{aligned} \frac{d\sigma_e}{d\Omega} &= \frac{d(N/N_0)}{d\Omega} \\ &= 6.77 \times 10^{-17} \times Z^{2/3} \cdot \left[ \sum_{i=1}^{i=3} \frac{a_i}{b_i^2 + \left( \frac{\theta}{\theta_0} \right)^2} \right] \quad \dots (2) \end{aligned}$$

where,

$$d\Omega = 2\pi\theta \cdot d\theta.$$

$$a_i = 0.1, \quad 0.55, \quad 0.35$$

$$b_i^2 = 36, \quad 1.44, \quad 0.09$$

$$\theta_0 = \frac{4.17 \times Z^{1/3}}{\phi_R^{1/2}},$$

and  $\theta$  is defined in figure 1. With the usual aperture diameters of 2.75, 5.5, 27.5, 55 and 275  $\mu$  and object to aperture distance of 2.75 mm, we get five values of  $\theta$  viz,  $5 \times 10^{-4}$ ,  $10^{-3}$ ,  $5 \times 10^{-3}$ ,  $10^{-2}$  and  $5 \times 10^{-2}$

$$\text{Substituting, } X = \left( \frac{\theta}{\theta_0} \right)^4,$$

$$\begin{aligned} \text{we get, } \sigma_e &= \int_{\theta}^{\infty} d\sigma_e \\ &= 4.255 \times 10^{-10} \times Z^{2/3} \times \frac{\theta_0^2}{2} \left[ \int_{\theta}^{\infty} \left( \sum \frac{a_i}{b_i^2 + x} \right)^2 dx \right] \\ &= \frac{3.699 \times 10^{-10} \times Z^{2/3}}{\phi_R} \times \left[ \int_{\theta}^{\infty} f(\theta, Z, \phi) \right] \dots (3) \end{aligned}$$

With the help of equation (3), the elastic scattering cross-sections have been calculated for U(92), Au(79), Pd(46), Cr(24) and C(6). For  $\phi = 60$  k v. and 5 values of  $\theta = 5 \times 10^{-4}$ ,  $10^{-3}$ ,  $5 \times 10^{-3}$ ,  $10^{-2}$ ,  $5 \times 10^{-2}$  radians which are frequently used in electron microscopic work. The calculated cross-sections are contained in Table I.

TABLE I

Calculated values of elastic scattering cross-sections ( $\sigma_e$ ) from Moliere's equation for different elements and for different aperture angles,  $\phi$  is assumed to be 60 k.v.

$\phi$ in k.v.	$\theta$ in radians	Elastic scattering cross section $\sigma_e$ in $10^{-18}$ cm <sup>2</sup>				
		C	Cr	Pd	Au	U
60	$5 \times 10^{-4}$	1.51	9.60	22.84	47.01	57.83
	$10^{-3}$	1.50	9.57	22.81	46.95	57.55
	$5 \times 10^{-3}$	1.26	8.87	21.69	45.31	55.73
	$10^{-2}$	0.89	7.36	18.99	41.05	50.94
	$5 \times 10^{-2}$	0.15	1.83	5.67	14.27	18.46

b) *Elastic Scattering According to Lenz.*

According to Lenz, the differential elastic scattering cross-section per unit solid angle is given by,

$$\frac{d\sigma_e}{d\Omega} = \frac{4Z^2}{a_R^2} \cdot \frac{1}{(\sigma^2 + 1/R^2)^2} \dots (4)$$

where,

$$d\Omega = 2\pi\theta \cdot d\theta$$

$$a_H = 0.528 \times 10^{-8} \text{ cm.}$$

$$R = a_H \cdot Z^{-1/3}.$$

$$q = \frac{2\pi\theta}{\lambda}$$

and,

$$\lambda = \frac{12.3 \times 10^{-8}}{\phi_R^{1/2}} \text{ cm.}$$

$$\therefore d\sigma_e = \frac{8\pi Z^2}{a_H^2} \cdot \frac{\theta d\theta}{(q^2 + 1/R^2)^2} \quad \dots (5)$$

Substituting,

$$q^2 = x, \quad 1.0., \quad \frac{4\pi^2\theta^2}{\lambda^2} = x; \quad \frac{4\pi^2}{\lambda^2} \cdot 2\theta d\theta = dx.$$

$$\therefore \theta \cdot d\theta = \frac{\lambda^2}{8\pi^2} \cdot dx.$$

$$\therefore d\sigma_e = \frac{8\pi Z^2}{a_H^2} \cdot \frac{\lambda^2}{8\pi^2} \cdot \frac{dx}{(x + 1/R^2)^2} \quad \dots (6)$$

$$\therefore \sigma_e = \int_{\theta}^{\infty} d\sigma_e = \frac{Z^2\lambda^2}{\pi a_H^2} \int \frac{dx}{(x + 1/R^2)^2}.$$

$$= \frac{Z^2\lambda^2}{\pi a_H^2} \cdot \left[ -\frac{1}{(x + 1/R^2)} \right]_{\theta}^{\infty} = \frac{Z^2\lambda^2}{\pi a_H^2} \left\{ \frac{1}{q^2 + 1/R^2} \right\} \quad \dots (7)$$

TABLE II.

Elastic scattering cross-sections ( $\sigma_e$ ) calculated from Lenz's equation for different elements and for different aperture angles,  $\phi$  is assumed to be 60 k.v.

$\phi$ in k.v.	$\theta$ in radians	Elastic scattering cross-section $\sigma_e$ in $10^{-18} \text{ cm}^2$				
		C	Cr	Pd	Au	U
60	$5 \times 10^{-4}$	0.83	5.25	12.40	25.71	31.50
	$10^{-3}$	0.83	5.25	12.49	25.70	31.49
	$5 \times 10^{-3}$	0.80	5.18	12.39	25.55	31.32
	$10^{-2}$	0.73	4.97	12.06	25.08	30.80
	$5 \times 10^{-2}$	0.18	2.20	6.58	15.80	20.11

With the help of equation (7), the elastic scattering cross-sections have also been calculated for the above elements for  $\phi = 60$  k.v. and different scattering angles. The calculated cross-sections are contained in Table II.

The results for  $\sigma_e$  by Moliere (Table I) and Lenz (Table II) are shown in figure 2. It has been found that the values of  $\sigma_e$  calculated from Moliere's equation

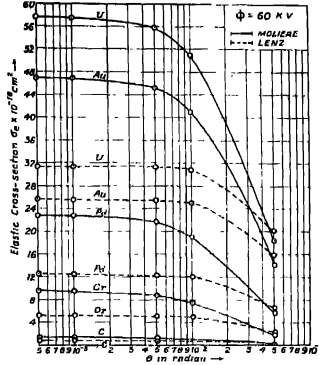


Fig. 2. Inelastic cross-section due to Lenz for 60 k.v. electrons as a function of  $\theta$  for different elements

are almost double than those of Lenz at small angles. The two values become nearly equal at an angle of  $5 \times 10^{-2}$  radians. The curves also indicate that for angles less than about  $5 \times 10^{-3}$  radians, the elastic cross-sections do not change appreciably with aperture angles.

### III. INELASTIC SCATTERING CROSS-SECTION

The atomic electrons will give rise to independent scattering as a result of which they are either raised to a higher level or ejected out of their orbits. The cross-section of this inelastic scattering has been shown by Lenz as,

$$\begin{aligned} \sigma_i &= \int_0^\infty d\sigma_i \\ &= \frac{2\lambda^2 R^2 Z}{\pi a_H^2} \cdot \left[ \frac{1}{2(q^2 R^2 + 1)} + \log_e \left( \frac{1}{1 + \frac{1}{q^2 R^2}} \right) \right]_q^\infty \\ &= \frac{2\lambda^2 R^2 Z}{\pi a_H^2} \cdot \left[ -\frac{1}{2(q^2 R^2 + 1)} - \log_e \left( \frac{1}{1 + \frac{1}{q^2 R^2}} \right) \right] \end{aligned}$$

$$= \frac{2\lambda^2 R^2 Z}{\pi \alpha_H^2} \cdot \left[ \log_e \left( 1 + \frac{1}{q^2 R^2} \right) - \frac{1}{2(q^2 R^2 + 1)} \right] \quad (8)$$

with the notations defined in equation (4).

With the help of equation (8), the cross-sections for inelastic scattering have been calculated for U, Au, Pd, Cr and C for 5 values of  $\theta$  and  $\phi = 60$  k.v. The calculated cross-sections are contained in Table III.

TABLE III

Calculated values of inelastic scattering cross-sections ( $\sigma_i$ ) for different elements for different aperture angles,  $\phi$  is assumed to be 60 k.v.

$\phi$ in k v.	$\theta$ in radians	Inelastic scattering cross section $\sigma_i$ in $10^{-18}$ cm <sup>2</sup>				
		C	Cr	Pd	Au	U
60	$5 \times 10^{-4}$	2.05	3.66	4.79	5.96	6.34
	$10^{-3}$	1.67	3.06	4.03	5.06	5.40
	$5 \times 10^{-3}$	0.80	1.66	2.29	2.98	3.20
	$10^{-2}$	0.46	1.08	1.56	2.09	2.27
	$5 \times 10^{-2}$	0.04	0.15	0.26	0.42	0.48

Comparing figs.(2) and (3) it will be noticed that the inelastic cross-sections

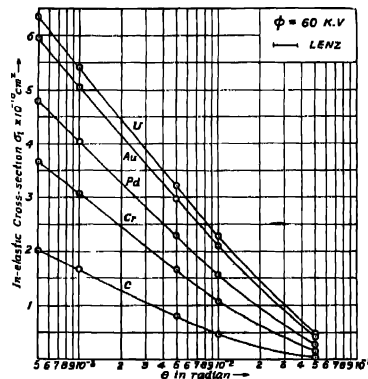


Fig. 3. Elastic and Inelastic cross-sections of Gold due to Moliere and Lenz for aperture angle  $10^{-2}$  radians as a function of beam potential  $\phi$

for the heavy metals Pd, Au and U are very small compared to the elastic cross-sections. Only in the case of C and Cr, the inelastic cross-sections are comparable to the elastic cross-sections.

#### IV. VARIATION OF SCATTERING CROSS-SECTION WITH BEAM POTENTIAL

With the help of equations (3), (7) and (8), the cross-sections for elastic and inelastic scattering have been calculated for Au, the metal most frequently employed for shadowing in electron microscopy for  $\theta = 10^{-2}$  radians and 4 values of  $\phi = 40, 60, 80$  and  $100$  k.v. The results are contained in Table IV, and are also plotted in figure 4 as a function of the beam potential  $\phi$ .

TABLE IV.

Calculated values of elastic ( $\sigma_e$ ) and inelastic ( $\sigma_i$ ) scattering cross-sections for Gold, for different beam potentials  $\phi$ ,  $\theta$  is assumed to be  $10^{-2}$  radians.

$\theta$ in radians	$\phi$ in k.v.	Scattering cross-section in $10^{-18} \text{ cm}^2$		
		$\sigma_e$ (Moliere)	$\sigma_e$ (Lenz)	$\sigma_i$ (Lenz)
$10^{-2}$	40	65.58	38.68	3.02
	60	41.05	25.08	2.09
	80	28.98	18.31	1.40
	100	21.89	14.26	1.01

It will be seen from figure 4 that for all voltages the inelastic scattering cross-section is negligible compared to elastic cross-section for gold.

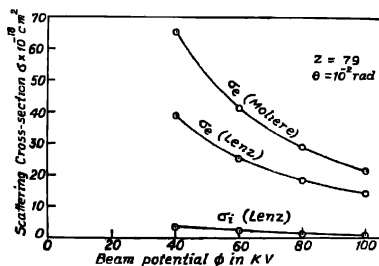


Fig. 4. Contrast diagram due to any thickness of C, Cr, Pd, Au and U—film— for 60 k.v. electrons and aperture angle  $10^{-2}$  radians, assuming Moliere's equation of elastic cross-section and Lenz's equation of inelastic cross-section.

#### V. CONTRAST IN ELECTRON MICROGRAPHS

A specimen of thickness  $t$  may be supposed to be divided into infinitesimal

layers of thickness  $dt$ , it can then be shown that if  $I_0$  be the incident beam intensity and  $I_t$  the transmitted intensity, we have the relation,

$$I_t = I_0 \cdot e^{-nt\sigma(\theta)} \quad \dots (9)$$

where  $\sigma(\theta)$  is the total cross-section per atom for scattering above a limiting angle  $\theta$ , and  $n$  the number of atoms per unit volume of the specimen.  $\theta$  is defined by the objective aperture as shown in figure 1.

The contrast  $g$  in the image is determined by the difference between the incident and the transmitted beam expressed as the fraction of the incident beam and can be expressed as

$$\begin{aligned} g &= (I_0 - I_t)/I_0 \\ &= 1 - e^{-nt\sigma(\theta)} \\ &= 1 - e^{-\frac{N}{A} \cdot \rho t \sigma(\theta)} \end{aligned} \quad (10)$$

TABLE V

Element and atomic number 'Z'	Film thickness 't' in A.U.	Contrast 'g' in percentage ( $\sigma_e$ by Moliere)	Contrast 'g' in percentage ( $\sigma_e$ by Lenz)
U(92)	5	11.89	7.57
	10	22.37	14.56
	15	31.60	21.02
	20	39.73	27.00
	25	46.90	32.52
	30	53.22	37.63
Au(79)	5	12.01	7.74
	10	22.57	14.89
	15	31.87	21.48
	20	40.05	27.56
	25	47.24	33.17
	30	53.58	38.34
Pd(46)	5	6.85	4.59
	10	13.24	8.98
	15	19.18	13.16
	20	24.72	17.16
	25	29.88	20.96
	30	34.68	24.59
Cr(24)	5	3.20	2.03
	10	6.28	4.54
	15	9.28	6.74
	20	12.17	8.89
	25	14.98	10.99
	30	17.69	13.03
C(6)	5	0.77	0.68
	10	1.52	1.34
	15	2.27	2.01
	20	3.02	2.66
	25	3.76	3.32
	30	4.50	3.97



where  $N$  = Avogadro number,  
 $A$  = Atomic weight  
 $\rho$  = Density of scattering substance in gm. cm<sup>-3</sup>  
 $t$  = Thickness in cm.

and,  $\sigma(\theta) = \sigma_s(\theta) + \sigma_i(\theta)$ , total integral scattering cross-section between  $\theta - \infty$ .

With the help of equation (10) and the previously calculated values of  $\sigma_s(\theta)$  and  $\sigma_i(\theta)$ , the contrasts corresponding to different film thicknesses are estimated taking both the elastic and inelastic scattering cross-sections into consideration. The results are contained in the Table V. Column 3 of this table shows the calculated values of contrasts based on the cross-section due to Moliere, while column 4 contains the values of contrast to be expected if Lenz equations are valid. Figures 5 and 6 show the contrast values for the elements most frequently used in shadow-casting. These diagrams are the plots of the results contained in Table V with  $g$  in per cent taken as abscissa and the corresponding film thickness in Angstroms taken as ordinate.

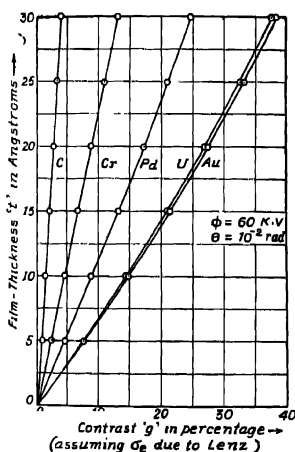


Fig. 5. Contrast diagram due to any thickness of C, Cr, Pd, Au and U—films for 60 k.v. electrons and aperture angle  $10^{-2}$  radians, assuming Lenz's equations of elastic and inelastic cross-sections.

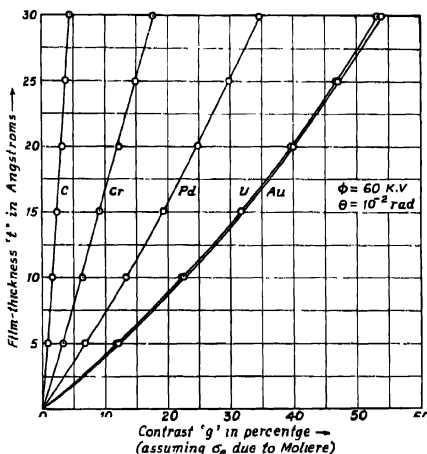


Fig. 6.

## VI. DISCUSSIONS

Contrast in the image of an electron micrograph is influenced, in addition to the scattering, by such factors as (a) the spherical aberration of the objective lens

which depends on the angular aperture of the objective, (b) voltage sensitivity of the photographic plate used in recording the image. For the lowest values of the aperture angles, the effect of aberration ( $\sim c_s \theta^3$ ) is negligible and if one decides to work at a fixed accelerating beam potential then obviously the effect of the beam potential on the photographic response need not be considered. So these two factors have been neglected in the present communication; only the effect of scattering of electrons by the specimen has been considered which is the main source of the production of contrast in electron micrograph.

In the light of the results presented in the paper, it is found that if 10% contrast is considered adequate for good representation, then the thicknesses of metals required are given in Table VI.

TABLE VI

Metal and atomic number 'Z'	Film thickness 't' in A.U.	
	from fig. (5)	from fig. (6)
Uranium—92	4.1	6.7
Gold—79	3.8	6.2
Palladium—46	9.8	11.0
Chromium—24	16.2	22.0

Hence from the accurate measurement of contrast with thin films of heavy elements at the lowest value of the aperture angle and at a fixed accelerating beam potential it should be possible to find the relative merits of the two formulations.

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